ARMY RESEARCH LABORATORY

Elastic Modulus by Resonance of Rectangular Prisms: Corrections for Edge Treatments

by George D. Quinn
and Jeffrey J. Swab

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Abstract

The dynamic elastic moduli of isotropic, homogeneous ceramics is commonly determined by resonance methods. A prismatic beam specimen is vibrated in a flexural mode, and the resonant frequency is measured. The beam may have a square, rectangular, or circular cross section. Elastic modulus is determined from the resonant frequency, the mass or density of the prism, and the beam's physical dimensions. Under ideal circumstances, the beam cross section should have a simple prismatic shape, but in practice, the method is sometimes applied to rectangular specimens with edge chamfers or radii that are applied to reduce edge flaw sensitivity during strength tests. The effect of such edge treatments on the resonance frequency and a simple means to correct the calculated elastic modulus for the edge treatment are provided in this note.

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1. Introduction

The dynamic elastic moduli of isotropic, homogeneous ceramics is commonly determined by resonance methods [1–6]. A prismatic beam specimen is vibrated in a flexural mode, and the resonant frequency is measured. The vibration may either be by continuous or impulse excitation. The beam may have a square, rectangular, or circular cross section. Elastic modulus is determined from the resonant frequency, the mass or density of the prism, and the beam's physical dimensions. Under ideal circumstances, the beam cross section should have a simple prismatic shape, but in practice, the method is sometimes applied to rectangular specimens with edge chamfers or radii that are applied to reduce edge flaw sensitivity during strength tests. The effect of such edge treatments on the resonance frequency and a simple means to correct the calculated elastic modulus for the edge treatment are provided in this note.

The basic wave equation for the propagation of an elastic wave in an elastic medium is

$$E = \rho v^2, \tag{1}$$

where E is the elastic modulus, ρ is the material density, and v is the wave speed. Goens [7] solved Timoshenko's [8] equation relating Young's modulus to the flexural resonance frequency for bars of different cross section. Pickett [3] further simplified Goen's solution for elastic modulus, E, which may be expressed in the following form:

$$E = C_1 W f^2, \tag{2}$$

where W is weight of the prism, f is the flexural resonant frequency, and C_1 is given by

$$C_1 = \frac{4\pi^2 \ell^3 T_1}{gI(4.730)^4}, \tag{3}$$

where ℓ is the prism length, g is the gravitational constant, I is the second moment of inertia for the beam cross section, and T_1 is a dimensionless geometric constant that depends upon the radius of gyration of the prism cross section, the length of the prism, and Poisson's ratio. Subsequent analysis and experimental work [1, 2, 4, 5] refined the equations for T_1 and led to an equation for E:

$$E_{b,\rho} = 0.9465 \frac{\rho \ell^4 f^2}{d^2} T , \tag{4}$$

where d is the specimen thickness and T is a new dimensionless geometric term. The subscripts b and ρ attached to E denote the formula is for an ideal rectangular beam (no edge treatment) and the calculation uses the density.

For an ideal rectangular beam (no edge treatment)

$$\rho_b = m/(bd\ell), \tag{5}$$

where ρ_b is the material density, m is the mass, and b is the specimen width. Substituting into equation 4, results in

$$E_{b,m} = 0.9465 mf^2 \frac{T}{b} \left(\frac{\ell^3}{d^3} \right), \tag{6}$$

where the subscripts b and m attached to E denote that the formula calculates the modulus of an ideal rectangular beam (no edge treatment) using the mass and physical dimensions of the beam. This latter form is commonly used today in standard test methods.

Empirical solutions for T_1 are available for ideal rectangular cross section prisms and are used in the ASTM flexural resonance standard test methods [9–14]. Several standards [9–12]

caution that chamfering or rounding of edges may create an experimental error of undefined magnitude. They recommend against the use of these bars, but this is unnecessarily restrictive as we will show.

The chamfers reduce the cross-sectional moment of inertia, I, and slightly alter the radius of gyration, and alter the relationship between density and the physical dimensions of the beam, equation 5. The effect upon I has previously been quantified in connection with work to minimize experimental error in flexure strength testing [15–17]. Even a small chamfer can alter I a meaningful amount and must be taken into account when preparing flexure specimens for strength testing. For example, a 45° chamfer of 0.15-mm size will reduce I by 1% for standard 3-mm × 4-mm cross section flexure strength specimens, which in turn causes the flexure stress to be underestimated by 1%. Consequently, the 0.15-mm chamfer size is the maximum allowed by several world flexure strength standards [18, 19]. Equations for I, for chamfered or round-edged beams in bending and error tables for the stress corrections, are available in the works of Baratta and coworkers [15, 16]. These equations for I are repeated here for convenience, and rather than list errors, a simple correction factor for the elastic modulus is furnished. The moment of inertia, I_b, for a rectangular cross section beam is

$$I_b = \frac{bd^3}{12}. \tag{7}$$

The true moment of inertia, I_t, for a beam with 45° chamfers of size c, as shown in Figure 1 [15, 16], is

$$I_t = \frac{bd^3}{12} - \frac{c^2}{9}\left(c^2 + \frac{1}{2}(3d - 2c)^2\right), \tag{8}$$

where the second term on the right-hand side shows the reduction due to the chamfers. It is assumed that the four chamfers are identical in size. The true moment of inertia, I_t, for a beam with four identical rounded edges of radius r, as shown in Figure 2 [16], is

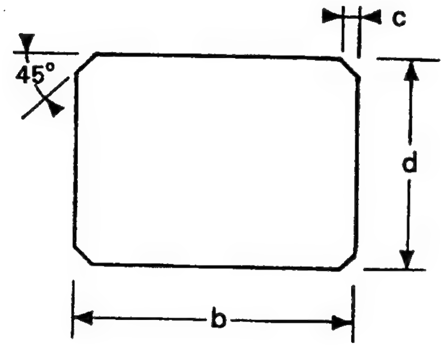

Figure 1. Specimen Cross Section for a Chamfered-Edge Beam.

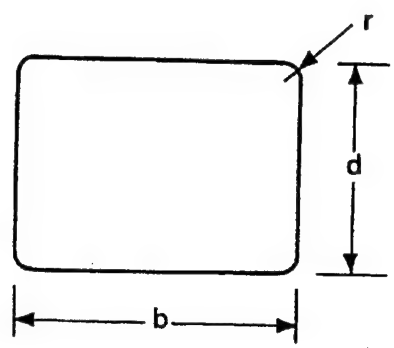

Figure 2. Specimen Cross Section for a Rounded-Edge Beam.

$$I_t = \frac{b(d-2r)^3}{12} + \frac{(b-2r)r^3}{6} + \frac{(b-2r)(d-r)^2 r}{2} + 4r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) + \pi r^2\left[\frac{d}{2} - r\left(1 - \frac{4}{3\pi}\right)\right]^2 . \quad (9)$$

Combining equations 2 and 3, the true elastic modulus, E_{cor}, may be calculated as follows:

$$\frac{E_{cor}}{E_b} = \frac{C_{1,cor} W f^2}{C_{1,b} W f^2} = \frac{I_b}{I_t}, \quad (10)$$

where E_b and $C_{1,b}$ are the calculated elastic modulus (using either equation 4 or 6) and constant C_1, respectively (assuming the specimen is a simple rectangular beam), uncorrected for chamfers

or edge rounding. $C_{1,cor}$ is the C_1 term corrected for the chamfering or edge rounding. The weight and frequency, f, are the values measured for the chamfered or edge-rounded beam. Then

$$E_{cor} = \left(\frac{I_b}{I_t}\right) E_b . \tag{11}$$

For standard specimens with a 3-mm × 4-mm cross section size,*

$$E_{cor} = FE_b , \tag{12}$$

where F† is the correction factor for the change in I that is due to the edge treatment and is given in Table 1 for various chamfers. Analogous values of F† for the same standard specimens with rounded edges of radius r are given in Table 2.

Equations 11 and 12, with moment of inertia correction only, should be used with equation 4 when the true density is known. The latter may be obtained from a water displacement measurement, or calculated from the mass and volume of the beam provided that the correction is made for the reduction in volume due to the edge treatment.

On the other hand, many standard test methods use equation 6, for which an assumption regarding the density, mass, and physical dimensions of the specimen (equation 5) has been invoked. If an edge treated beam is used, then an additional correction to remedy this assumption should be made as follows. The correct density, ρ_T, of a chamfered beam is

$$\rho_T = m/\ell\left(bd - 2c^2\right), \tag{13}$$

---

* The same factor may be applied to correct the flexure stress, $\sigma_t = F\sigma_r$, where σ_t is the true, maximum flexure stress in a chamfered or rounded beam and σ_b is the apparent flexure stress assuming a rectangular cross section.

† The adjustments listed in Tables 1 and 2 are applicable only for <u>flexural</u> modes of resonance and are not appropriate for the longitudinal or torsional resonance modes.

Table 1. Correction Factors, F and P, for Chamfered Standard 3-mm × 4-mm Strength Test Specimens[a]

c (mm)	Moment Correction Factor, F b = 4 mm, d = 3 mm	Density Correction Factor, P b = 4 mm, d = 3 mm
0.080	1.0031	1.0011
0.090	1.0039	1.0014
0.100	1.0048	1.0017
0.110	1.0058	1.0020
0.115	1.0063	1.0022
0.118	1.0066	1.0023
0.120	1.0069	1.0024
0.122	1.0071	1.0025
0.124	1.0073	1.0026
0.126	1.0076	1.0027
0.128	1.0078	1.0027
0.130	1.0080	1.0028
0.132	1.0083	1.0029
0.134	1.0085	1.0030
0.136	1.0088	1.0031
0.138	1.0090	1.0032
0.140	1.0093	1.0033
0.150	1.0106	1.0038
0.160	1.0121	1.0043
0.170	1.0136	1.0048
0.180	1.0152	1.0054
0.190	1.0169	1.0061
0.200	1.0186	1.0067
0.210	1.0205	1.0074
0.220	1.0224	1.0081
0.230	1.0244	1.0089
0.240	1.0265	1.0097
0.250	1.0287	1.0105

[a] A chamfer size of 0.150 mm is the maximum value allowed for this geometry by ASTM C1161 and ISO 17404.

Table 2. Correction Factors, F and P, for Edge Rounded Standard 3-mm × 4-mm Strength Test Specimens[a]

r (mm)	Moment Correction Factor, F b = 4 mm, d = 3 mm	Density Correction Factor, P b = 4 mm, d = 3 mm
0.080	1.0013	1.0005
0.090	1.0017	1.0006
0.100	1.0021	1.0007
0.110	1.0025	1.0009
0.120	1.0030	1.0010
0.130	1.0035	1.0012
0.140	1.0041	1.0014
0.150	1.0046	1.0016
0.160	1.0053	1.0018
0.170	1.0059	1.0021
0.180	1.0066	1.0023
0.190	1.0074	1.0026
0.200	1.0082	1.0029
0.210	1.0090	1.0032
0.220	1.0098	1.0035
0.230	1.0107	1.0038
0.240	1.0116	1.0041
0.250	1.0126	1.0045
0.260	1.0136	1.0049
0.270	1.0146	1.0052
0.280	1.0157	1.0056
0.290	1.0168	1.0061
0.300	1.0180	1.0065

[a] A rounded edge of 0.200 mm is the maximum value allowed for this geometry by ASTM C1161 and ISO 14704.

and for an edge rounded beam,

$$\rho_T = m/\ell\left(bd - r^2\,(4 - \pi)\right), \qquad (14)$$

and then

$$E_{cor} = \left(\frac{I_b}{I_t}\right)\left(\frac{\rho_T}{\rho_b}\right)E_{b,m}, \qquad (15)$$

where $E_{b,m}$ is from equation 6, which assumes the beam is an ideal rectangle. For standard 3-mm × 4-mm cross section specimens, the corrected modulus is

$$E_{cor} = FPE_{b,m}, \tag{16}$$

where P is the correction factor for the change in ρ due to the edge treatment. Values of P for standard 3-mm × 4-mm specimens with either chamfered or rounded edges are listed in Tables 1 and 2, respectively.

2. Experimental Procedure

Four ceramic materials listed in Table 3 were used to examine the effect of edge chamfering on the resonant frequency. Rectangular specimens were prepared with a chamfer geometry as depicted in Figure 1. Three of the four materials had average chamfer sizes (Table 3), which are well over the 0.15-mm tolerance commonly specified in flexure strength standards. The resonant frequency of each material was measured with a commercial impulse excitation instrument.* The resonant frequency typically was measured three to five times for each specimen and was repeatable to within 0.01 kHz. The specimen cross section dimensions were measured with a hand micrometer with a resolution and precision of 0.002 mm. Some specimens may have had a slight thickness taper (~0.002 mm) along the length, so the cross section dimensions were measured in the middle of the beam. Length was measured with a hand caliper with a resolution of 0.01 mm. Mass was measured with a precision laboratory balance to within 0.001 g, and the density was determined using the mass and physical dimensions of the specimen. The elastic modulus was calculated using equation 4. For a perfectly rectangular beam, the uncertainty of the elastic modulus may be estimated from a propagation of uncertainties of the individual variables in equation 4 [20]. Using the instrument resolutions and precisions listed previously, the type B (95% confidence limit) uncertainties for mass, width, thickness, length, and frequency

* Grindosonic Mk5, J. W. Lemmens, Inc., St. Louis, MO.

Table 3. $E_{b,m}$ and E_{cor} Values for Ceramic Materials

Material[a]	Density (ρ_T)[f] (g/cm^3)	Average Frequency (kHz)	Average $E_{b,m}$ (GPa)	Average c (mm)	Moment (I) Correction Factor (F)	E_{cor} [I Only] (GPa)	Density (ρ) Correction Factor (P)	E_{cor} [ρ and I] (GPa)
sintered Al_2O_3[b]	3.956 ± 0.003	11.02	384.2 ± 0.6	0.230	1.0258[g]	394.1 ± 0.6	1.0095	397.8 ± 0.6
hot-pressed SiC[c]	3.206 ± 0.003	14.29	445.9 ± 1.4	0.191	1.0170	453.5 ± 1.5	1.0061	456.2 ± 1.5
hot-pressed TiB_2[d]	4.392 ± 0.008	12.83	488.4 ± 3.6	0.132	1.0083	492.5 ± 3.8	1.0029	493.9 ± 3.6
AlON[e]	3.644 ± 0.006	11.65	312.0 ± 1.6	0.198	1.0183	317.7 ± 1.4	1.0066	319.8 ± 1.4

[a] Certain commercial materials or equipment are identified in this report to adequately specify the experimental procedure. Such identification does not imply endorsement by the National Institute of Standards and Technology or the U.S. Army Research Laboratory nor does it imply that these materials or equipment are necessarily the best for the purpose.

[b] Grade AD-999, Coors, Golden, CO.

[c] Grade SiC-N, Cercom Inc., Vista, CA.

[d] Cercom Inc., Vista, CA.

[e] Raytran Aluminum Oxynitride, Raytheon Company, Lexington, MA.

[f] Calculated from mass and physical dimensions of unchamfered specimens, or in the case of alumina, from chamfered specimens with correction.

[g] Since the alumina specimens did not have 3-mm × 4-mm cross sections, F was obtained using equations 8 and 11 and <u>not</u> from Table 1.

Notes: Uncertainties are ±1 standard deviation based upon scatter of the individual outcomes from 3–5 specimens per material.

are 0.042%, 0.050%, 0.067%, 0.020%, and 0.091%, respectively. The total uncertainty in E is 0.29%. The chamfer sizes were measured with a binocular stereomicroscope at magnifications of up to 160× in conjunction with a precision traversing stage with micrometer heads with a digital readout of 0.001-mm resolution. All four chamfers were measured. There was some variability in chamfer sizes for a given specimen, but only an average value for each specimen was used for the purpose of correcting the elastic modulus.

3. Results and Discussion

Table 3 summarizes the measured (uncorrected) elastic modulus, E_b, values determined from these frequencies and the elastic modulus values corrected for density and the edge chamfering, E_{cor}.

Three sintered alumina specimens were 2.816 mm × 4.006 mm × 50.7 mm in size, close to the standard size of 3 mm × 4 mm × 45 mm - 50 mm. The resonant frequencies of the three bars were nearly identical: 10.99 kHz, 11.06 kHz, and 11.00 kHz. The uncorrected elastic moduli averaged 384.2 GPa. This was corrected, for I only, by 2.58% for the finite chamfer size of 0.230 mm to 394.1 GPa, a value in excellent agreement with 395 GPa measured by an ultrasonic time-of-flight method on the same batch of material as shown in Table 4. (Since the alumina specimens did not have standard 3-mm × 4-mm cross sections, the moment correction factor, F, was obtained using equations 8 and 11, and **not** from Table 1.) The correction for the true density increased E_{cor} by an additional 0.95% to 397.8 GPa. This value is good in agreement with the ultrasonic time of flight value. A single additional chamfered specimen was strain gauged and tested in a semiarticulting four-point flexure fixture. The static elastic modulus calculated from the static strains was 386.9 GPa, which is slightly higher than the uncorrected dynamic E but lower than the correct dynamic E. Static elastic modulus and strain gauge uncertainties may be of the order of several percent [4, 21], which may account for the discrepancy between the static and dynamic E values.

Table 4. Comparison of Corrected Beam Resonance Elastic Moduli, E_{cor}, to Values From Other Methods

Material	E resonance, chamfered, [corrected for I only] (GPa)	E resonance, chamfered, [corrected for I and P] (GPa)	E resonance, unchamfered (GPa)	E ultrasonic, time of flight (GPa)	E resonance disks (GPa)
sintered Al_2O_3	394.1	397.8	—	395	—
hot-pressed SiC	453.5	456.2	453.5	—	—
hot-pressed TiB_2	492.5	493.3	491.9	—	—
AlON	317.7	319.8	317.7	—	316.7

Notes: — = Not measured.

Flexure specimens of the three other ceramic materials were prepared, both with and without edge chamfering. Most of the bars had nominal dimensions of 3 mm × 4 mm × 50 mm, with the sole exception of the SiC specimens without edge chamfering, which were slightly larger, having nominal dimension of 3.8 mm × 4 mm × 51 mm. Ten specimens, 5 with chamfering and 5 without, were examined for the each of the TiB_2 and AlON materials, while 8 specimens, 4 with chamfering and 4 without, were examined for the SiC.

The average resonant frequencies measured for the SiC, TiB_2 and AlON, with chamfers resulted in uncorrected average elastic moduli values, E_b, of 445.9 GPa, 488.4 GPa, and 312.0 GPa, respectively. These values were then corrected, for I only, by 1.70%, 0.83%, and 1.83%, respectively, based on the average chamfer sizes, which ranged from 0.13 mm to 0.20 mm (Table 3). The corrected values compare exceptionally well with computed elastic moduli for specimens *without chamfers*, as shown in Table 4. As in the alumina instance, the second correction for the true density increased E_{cor} for all three of these material. When compared to the results from the unchamfered bars and the other methods, these values are still in good agreement, but they do not agree as well as the values corrected for I only.

In the case of the AlON, comparable resonance values were available from testing 10 disks, nominally 50 mm in diameter × 8.3 mm thick, determined in accordance with ASTM C 1259 [11]. The average elastic modulus was 316.7 GPa, in excellent agreement with the beam resonance results (Table 4) from unchamfered bars and from chamfered bars corrected for I only.

4. Summary

In summary, the mathematical solutions to account for the effect of edge chamfers on the density and moment of inertia, and in turn the dynamic elastic modulus, of a rectangular ceramic beam has been experimentally verified. The analysis and experimental results show that change in the moment of inertia, due to edge chamfering, has a greater impact on the resultant elastic modulus of a rectangular ceramic beam than the change in density. For standard 3 mm × 4 mm cross section beams Tables 1 and 2 provide a simple means to correct elastic moduli values for the change in moment of inertia and density due to edge chamfering or rounding. For beams with nonstandard cross sections, equation 11 or 15 should be used, depending on the type of edge treatment and whether the true density is available.

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6. AUTHOR(S)
George D. Quinn* and Jeffrey J. Swab

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-WM-MC
Aberdeen Proving Ground, MD 21005-5069

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*National Institute of Standards and Technology, Ceramics Division, Gaithersburg, MD 20899

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The dynamic elastic moduli of isotropic, homogeneous ceramics is commonly determined by resonance methods. A prismatic beam specimen is vibrated in a flexural mode, and the resonant frequency is measured. The beam may have a square, rectangular, or circular cross section. Elastic modulus is determined from the resonant frequency, the mass or density of the prism, and the beam's physical dimensions. Under ideal circumstances, the beam cross section should have a simple prismatic shape, but in practice, the method is sometimes applied to rectangular specimens with edge chamfers or radii that are applied to reduce edge flaw sensitivity during strength tests. The effect of such edge treatments on the resonance frequency and a simple means to correct the calculated elastic modulus for the edge treatment are provided in this note.

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